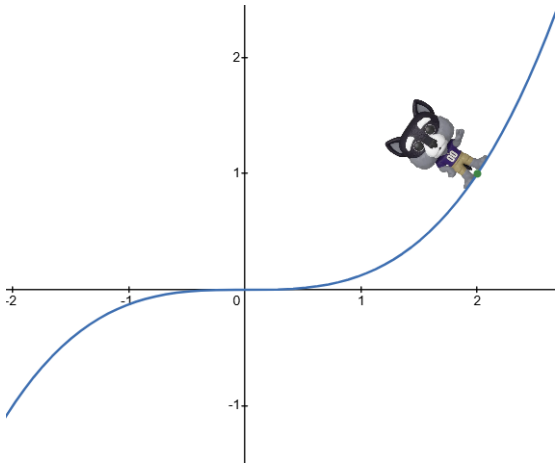
14.5 The Chain Rule

Recall: The Chain Rule from Calculus 1

$$\frac{dy}{dt} = \frac{dy}{dx}\,\frac{dx}{dt}$$



https://www.desmos.com/calculator/omcdt4vltj

Example:

Find $\frac{dy}{dt}$ in two ways.

$$y = \frac{1}{8}x^3, \quad x = t^2 - 2$$

(a) Substitute first.

(b) Use the chain rule.

Case 1: Two inputs depending on time

Given $z = f(x,y), x = x(t), y = y(t)$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

Example:

$$z = xy^2 - x^3y, \qquad x = t^2, y = t^3$$

Find $\frac{dz}{dt}$ when t = 1 in two ways

(a) Substitute first.

(b) Use the chain rule

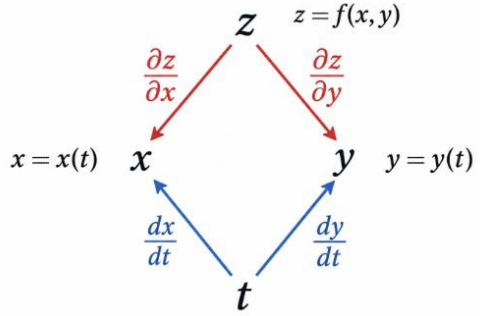
Note 1: Connects to linear approximation

$$\Delta z \approx f_x \, \Delta x + f_y \, \Delta y$$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

Note 2: Chain Rule Tree Diagram

multiply along branches and add results.



Case 2: Two independent variables HW Q2

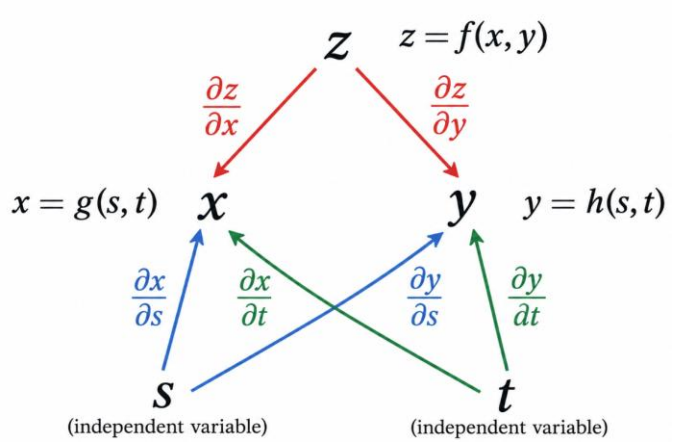
$$z = f(x, y), \quad x = x(s, t), \quad y = y(s, t)$$

$$\frac{\partial z}{\partial s} = f_x x_s + f_y y_s$$

$$\frac{\partial z}{\partial t} = f_x x_t + f_y y_t$$

Note: Chain Rule Tree Diagram

multiply along branches and add results.



Example:

$$z = 2x^2 - 2y^2, \quad x = s^2 - t, \quad y = s - t^3$$

Find $\frac{dz}{ds}$ and $\frac{dz}{dt}$ when t = 1 and s = 1

Chain Rule with Given Rates HW Q3–4

$$p(t) = f\big(g(t), h(t)\big)$$

Sometimes we know the derivatives of f at a point, but not a formula for f.

$$g(5) = 8,\ h(5) = 3,$$

$$g'(5) = -5,\ h'(5) = 4$$

$$f_x(8,3) = 5, \quad f_y(8,3) = 3$$

Find $p'(5)$.

Implicit Differentiation Revisited

Consider the surface

$$x^2 + 2y^2 + 9z^2 = 1$$

(a) Find $\frac{\partial z}{\partial x}$ using implicit differentiation.

(b) Now consider

$$w = x^2 + 2y^2 + 9z^2 - 1$$

Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial z}$.

Note: *Implicit Shortcut* **-** Given an implicit equation, get everything to one side and differentiate with respect to each variable separately and use…

$$F(x, y, z) = 0 \Rightarrow z_x = -\frac{F_x}{F_z},\ z_y = -\frac{F_y}{F_z}$$

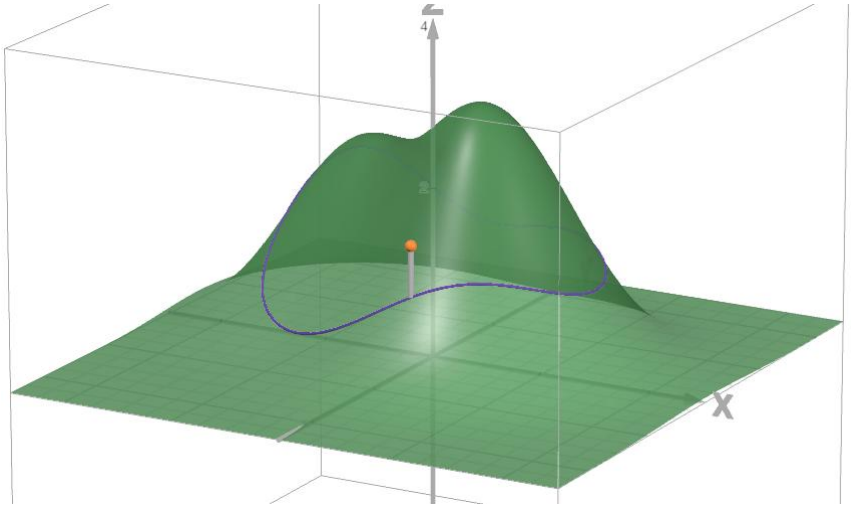
Optional: Walking Across a Landscape

A walker follows a path across the surface

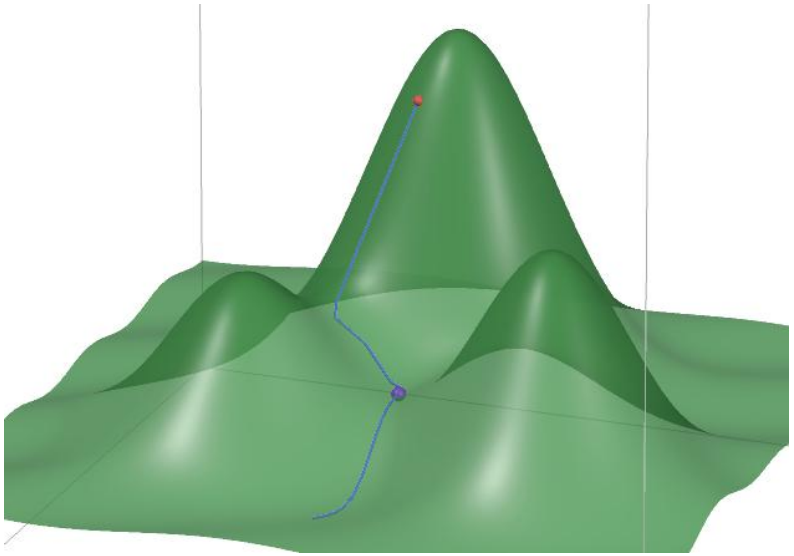
$$z = f(x, y).$$

$$\mathbf{r}(t) = \langle x(t), y(t), f\big(x(t), y(t)\big)\rangle$$

How quickly is the walker gaining or losing elevation?



Optional: A River Thru a Mountain



A river is traveling down a mountain range defined by $z = f(x, y)$.

No matter how it meanders down, what must be true about the change in height with respect to the river's length?

Meaning what can you say about

$$\frac{dz}{ds}$$